A Method for Obtaining Aerodynamic Coefficients from Yawsonde and Radar Data

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A numerical integration method for the reduction of yawsonde data from instrumented projectiles in free flight has been developed. Yawsonde data represent two-dimensional information (solar aspect angle and spin) while the actual motion is three dimensional (pitch, yaw, and spin). The method obtains a best fit of the aerodynamic coefficients in the equations of motion using numerical integration. The coefficients are iteratively adjusted in the manner developed by Chapman and Kirk. Trajectory information is derived from radar track data and is reduced to provide axial force coefficients and Earth-fixed velocity components. The trajectory data reduction uses the same numerical integration method as the angular data reduction. The results of the trajectory or swerve reduction are used as inputs to the angular reduction. Linear aerodynamic coefficients have been derived from the flight of an M549, 155mm rocket assisted projectile. Values of nonlinear aerodynamic coefficients have also been obtained using the numerical integration method but their validity remains to be proven. Both the angular and the trajectory reductions include the effects of wind.

Nomenclature

A	= reference area
C_{lp}	= spin deceleration coefficient
C_m	= pitching moment coefficient
C_{mq}	= damping in pitch coefficient
C_{np}	= Magnus moment coefficient
C_D	= drag coefficient
C_N	= normal force coefficient
C_X	= axial force coefficient
C_{Yp}	= Magnus force coefficient
L	= reference diameter
g	= acceleration due to gravity
I_x	= axial moment of inertia
I	= transverse moment of inertia
m	= mass
p	= spin rate
$\bar{q}$	= dynamic pressure
t_i	= direction cosines of solar vector in Earth-fixed coordinate system ($i = 1, 2, 3$)
u, v, w	= velocity components along missile x, y, z axes, respectively
V	= total missile velocity
V_{wxe}, V_{wye}	= components of wind velocity along Earth x, y, axes, respectively
V_{xe}, V_{ye}, V_{ze}	= velocity components along Earth x, y, z axes, respectively
x_e, y_e, z_e	= Earth-fixed axes
$\bar{\alpha}$	= total angle of attack
δ_e, γ_e	= trajectory elevation, azimuth, respectively
ε	= $\sin \bar{\alpha}$
$\theta_{fp}, \psi_{fp}, \phi_{fp}$	= fixed plane missile orientation angles
ρ	= density
σ	= solar aspect angle

Superscripts

$(\dot{\ })$	= differentiation with respect to time
$(\ddot{\ })$	= second derivative with respect to time

Subscripts

v	= velocity dependent
w	= wind condition
α	= force or moment coefficient slope

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Introduction

DURING the past several years yawsondes have been used to measure inflight dynamical behavior of projectiles over long flight paths.[1] The yawsonde produces large volumes of data on the solar aspect angle and spin of the shell during flight. Yawsonde data are two-dimensional. The angle between the projectile axis and the sun as well as the spin are determined along the flight path. The actual angular motion of the shell, however, is three-dimensional consisting of pitch, yaw, and spin. The basic problem is to extract the three-dimensional behavior of the shell from two-dimensional data. Additional information on the trajectory and meteorological conditions are required as inputs.

The conventional method for reducing data from free-flight tests is to fit solutions to the equations of motion to the range data.[2] Such procedures assume that the equations of motion can be solved in closed form or in an approximate way (quasi-linear). The conventional method can be applied to yawsonde data using a technique called "wobble."[3] A modified version of "wobble" has been used with limited success.[1] Still another method, developed by Haseltine,[4] uses an analog computer set up with the equations of motion. The operator adjusts the coefficients of the equations until the motion predicted by the analog computer agrees with the flight data.

Chapman and Kirk[5] recently developed a method for fitting differential equations of motion directly to the data using a unique approach for adjusting the coefficients of the equations. This method does not require that the equation of motion have an analytical solution. Whyte and others have investigated the method of Chapman and Kirk and applied it to data from aerodynamic spark range firings with excellent results.[6]

In this paper it will be shown that the numerical integration technique can be applied to yawsonde data to obtain linear aerodynamic coefficients over extended portions of the flight of a single projectile. Actual reductions were performed on a variety of projectile types. Here the discussion is limited to the work done on the 155mm, M549 rocket assisted projectile. This projectile was chosen so that direct comparisons could be made with the results of aerodynamic range and wind-tunnel tests.

This paper will discuss the aerodynamic equations for angular and swerving motion and the numerical integration method itself. The results of axial force and angular motion reduction will be presented and compared with linear theory reductions of range and wind-tunnel tests. The effect of including wind and velocity dependent coefficients in the method is described and discussed.

Theoretical Considerations

Linearized theory has been used to describe the motion of symmetric missiles with outstanding success during the past two decades. The theory is, however, restricted to small angle motion with small velocity and spin changes and nearly linear aerodynamic coefficients. Nonlinear coefficients may be determined by linear theory from overlapping fits if sufficient data are available or if sufficient number of range tests of the same projectile are made.

None of the above restrictions apply to the method of Chapman and Kirk. This method is conceptually and practically attractive since aerodynamic coefficients can be determined by fitting the differential equations of motion directly to the observed angular motion data. The force coefficients can be obtained by fitting the equations of swerving motion directly to positional data. The general method can be summarized briefly as follows. The equations of motion appropriate to the behavior of the projectile are numerically integrated using estimated values of the coefficients and the initial conditions. The same equations of motion are partially differentiated on each coefficient forming a set of parametric equations. The parametric equations are then numerically integrated to obtain values of the partial derivatives with respect to each coefficient. A differential corrections equation is set up from a Taylor expansion of the dependent variables of the equations of motion. The data are then compared to the computed values of the dependent variables in a least squares sense and corrections to the coefficients are obtained from the differential corrections equations. The coefficients of the equations of motion are adjusted by these corrections and the process begins again with new values of coefficients. The method iterates until convergence is achieved.

Equations of Motion

The equations of motion of a symmetric missile have been derived in a fixed plane coordinate system.[7] The rotation of the Earth is ignored and the aerodynamic coefficients are assumed to have the form of a power series (as functions of $\sin\bar{\alpha}$).

$$C_{m_\alpha} = a_0 + a_3\varepsilon^2 + a_5\varepsilon^4 + a_v(V - V_0) \tag{1}$$

$$C_{n_{p\alpha}} = b_0 + b_3\varepsilon^2 + b_5\varepsilon^4 \tag{2}$$

$$C_{m_q} = d_0 + d_2\varepsilon^2 \tag{3}$$

The static and Magnus coefficient expansions include fifth-order coefficients and may even, in some cases, require higher order terms. The motion of the center of mass of the projectile is described by the force equations (4–6).

$$\dot{u} = -\bar{q}\,AC_x/m + g\sin\theta_{fp} - \dot{\theta}_{fp}\,w + \dot{\psi}_{fp}\cos\theta_{fp}\,v \tag{4}$$

$$\dot{v} = -\bar{q}\frac{A}{m}C_{N_\alpha}\frac{v_w}{V_w} + \bar{q}\frac{A}{m}\frac{pL}{2V_w}C_{Y_{p\alpha}}\frac{w_W}{V_w} - \dot{\psi}_{fp}\cos\theta_{fp}[u + w\tan\theta_{fp}] \tag{5}$$

$$\dot{w} = -\bar{q}\frac{A}{m}C_{N_\alpha}\frac{w_w}{V_w} - \bar{q}\frac{A}{m}\frac{pL}{2V_w}C_{Y_{p\alpha}}\frac{v_w}{V_w} - g\cos\theta_{fp} + \dot{\psi}_{fp}\,v\sin\theta_{fp} + \dot{\theta}_{fp}\,u \tag{6}$$

The angular motion of the projectile is described by the moment equations (7–9).

$$\ddot{\phi}_{fp} = \bar{q}\frac{AL}{I_x}\frac{pL}{2V_w}C_{l_p} + \ddot{\psi}_{fp}\sin\theta_{fp} + \dot{\theta}_{fp}\dot{\psi}_{fp}\cos\theta_{fp} \tag{7}$$

where the spin is

$$p = \dot{\phi}_{fp} - \dot{\psi}_{fp}\sin\theta_{fp}$$

$$\begin{aligned}\ddot{\theta}_{fp} = \bar{q}\frac{AL}{I}&[a_0 + a_3\varepsilon^2 + a_5\varepsilon^4 + a_v(V - V_0)]w_w/V_w + \\ &(\bar{q}AL^2/2V_wI)[d_0 + d_2\varepsilon^2]\,\dot{\theta}_{fp} + \\ &\bar{q}(AL^2p/2V_wI)[b_0 + b_3\varepsilon^2 + b_5\varepsilon^4](v_w/V_w) - \\ &\dot{\psi}_{fp}\,p(I_x/I)\cos\theta_{fp} - \dot{\psi}_{fp}^2\cos\theta_{fp}\sin\theta_{fp}\end{aligned} \tag{8}$$

$$\begin{aligned}\ddot{\psi}_{fp} = [&-\bar{q}(AL/I)[a_0 + a_3\varepsilon^2 + a_5\varepsilon^4 + a_v(V - V_0)]v_w/V_w + \\ &\bar{q}(AL^2/2V_wI)[d_0 + d_2\varepsilon^2]\,\dot{\psi}_{fp}\cos\theta_{fp} + \\ &\bar{q}(AL^2p/2V_wI)[b_0 + b_3\varepsilon^2 + b_5\varepsilon^4]w_w/V_w + \\ &2\,\dot{\theta}_{fp}\,\dot{\psi}_{fp}\sin\theta_{fp} + \dot{\theta}_{fp}p(I_x/I)]/\cos\theta_{fp}\end{aligned} \tag{9}$$

θ_{fp} and ψ_{fp} are angles which define missile orientation in the Earth-fixed plane coordinate system. θ_{fp} describes a rotation about the missile y axis and ψ_{fp} is a rotation about the Earth z axis. The linear motion of the projectile is described by the momentum equations (10–12) in an Earth-fixed coordinate system.

$$\begin{aligned}\dot{V}_{xe} = &-\bar{q}(A/m)[C_{x0} + C_{x2}\,\varepsilon^2]\cos\theta_{fp}\cos\psi_{fp} + \\ &\bar{q}(A/m)C_{N_\alpha}[(v_w/V_w)\sin\psi_{fp} - (w_w/V_w)\sin\theta_{fp}\cos\psi_{fp}] - \\ &(\bar{q}A/m)(pL/2V_w)C_{Y_{p\alpha}}[(w_w/V_w)\sin\psi_{fp} + v_w/V_w\sin\theta_{fp}\cos\psi_{fp}] - \\ &(\bar{q}A/m)[C_{x_v}(V_w - V_0)]\cos\theta_{fp}\cos\psi_{fp}\end{aligned} \tag{10}$$

$$\begin{aligned}\dot{V}_{ye} = &-\bar{q}(A/m)[C_{x0} + C_{x2}\varepsilon^2]\cos\theta_{fp}\sin\psi_{fp} - \\ &\bar{q}(A/m)C_{N_\alpha}[(v_w/V_w)\cos\psi_{fp} + \\ &(w_w/V_w)\sin\theta_{fp}\sin\psi_{fp}] + \\ &\bar{q}(A/m)(pL/2V_w)C_{Y_{p\alpha}}[(w_w/V_w)\cos\psi_{fp} - \\ &(v_w/V_w)\sin\theta_{fp}\sin\psi_{fp}] - \\ &(\bar{q}A/m)[C_{x_v}(V_w - V_0)]\cos\theta_{fp}\sin\psi_{fp}\end{aligned} \tag{11}$$

$$\begin{aligned}\dot{V}_{ze} = &\bar{q}\frac{A}{m}[C_{x0} + C_{x2}\varepsilon^2]\sin\theta_{fp} - \bar{q}\frac{A}{m}C_{N_\alpha}\frac{w_w}{V_w}\cos\theta_{fp} - \\ &\bar{q}\frac{A}{m}\frac{pL}{2V_w}C_{Y_{p\alpha}}\frac{v_w}{V_w}\cos\theta_{fp} + \frac{\bar{q}A}{m}[C_{x_v}(V_w - V_0)]\sin\theta_{fp} - g\end{aligned} \tag{12}$$

where

$$V_w = (V_{xe_{(w)}}{}^2 + V_{ye_{(w)}}{}^2 + V_{ze_{(w)}}{}^2)^{1/2}, \quad \text{and} \quad \bar{q} = \tfrac{1}{2}\rho\,V_w^2$$

The axial force coefficient has been expanded as

$$C_x = C_{x0} + C_{x2}\varepsilon^2 \tag{13}$$

and the normal and Magnus force coefficients can be expanded

$$C_{N_\alpha} = n_0 + n_3\,\varepsilon^2 \tag{14}$$

$$C_{Y_{p\alpha}} = m_0 + m_3\,\varepsilon^2 \tag{15}$$

A Mach number dependent axial force coefficient, C_{x_v}, has been included in Eqs. (10–12) since large velocity changes are often encountered in flight data.

Time is the independent variable in the above equations rather than distance along the trajectory (arc length). Distance is a more natural base for data taken in aerodynamic spark ranges, while time is the natural variable for yawsonde and radar data. The method of Chapman and Kirk is now used in a computer program called "heeve" to best fit Eqs.

(10–12) to the radar position-time data. Estimated values of θ and ψ are used to obtain fitted values of position, velocity, axial force, normal force, and Magnus force. Before the angular motion equations (7–9) can be fitted to the yawsonde data, a relationsip between the solar aspect angle and the angles θ_{fp}, ψ_{fp} must be derived.

Solar Aspect Angle Relation

Yawsonde data give one-dimensional angular motion of the missile with respect to the sun. The solar aspect angle, σ, is the angle between the longitudinal axis of the projectile and a vector directed from the projectile to the sun. Let t_1, t_2, t_3 be the direction cosines of the solar vector in an Earth-fixed coordinate system (Fig. 1). θ_{fp} and ψ_{fp} locate the axis of the projectile in the same corodinate system. The solar aspect angle is obtained from the dot product of a unit solar vector and a unit vector along the missile axis

$$\cos\sigma = t_1 \cos\theta_{fp} \cos\psi_{fp} + t_2 \cos\theta_{fp} \sin\psi_{fp} + t_3 \sin\theta_{fp} \qquad (16)$$

Equation (16) provides the required relationship between the solar aspect angle, the orientation of the sun, and the angles of the missile axis relative to the Earth.

The angular motion of the projectile is fitted to the angular equations (7–9) in a least squares sense by comparing computed values of σ using Eq. (16) to the measured values of σ. The method of Chapman and Kirk is used to adjust the coefficients of the angular motion equations until the probable error of fit is a minimum.

Yawsonde Reduction Procedure

In order to obtain aerodynamic coefficients from yawsonde flight data, the c.m. motion of the projectile must first be well determined. Measurements of the physical properties of the shell are taken before firing and meteorological data on density, temperature, and wind velocities are noted at the time of firing. First a reduction on the position time data is made to obtain a best description of the trajectory. The results of this reduction are used as inputs to the angular motion reduction. If necessary, the results of the angular motion reduction could be fed back into the position-time reduction for improved accuracy.

The sequence of data reduction using the numerical integration technique is as follows. First the raw radar position-time data are reduced to obtain a drag profile. The inputs to this swerve reduction are measured values of position (x_e, y_e, z_e) and time, physical properties of the projectile, meteorological data on winds, denisty and temperature, and the direction cosines of the solar vector in an Earth-fixed coordinate system.

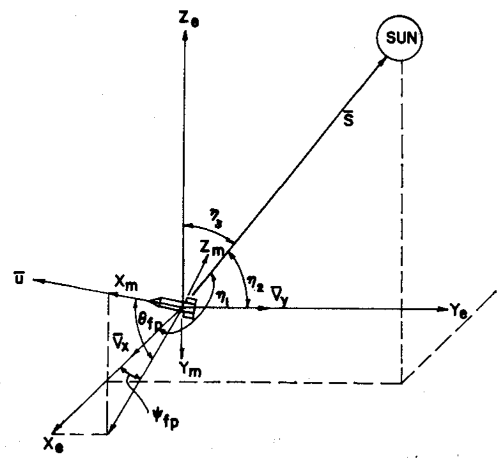

Fig. 1 Geometric relationship of the missile axis to the solar vector in a trajectory coordinate system.

Table 1 Reduction inputs and outputs

	Swerve reduction	Angular reduction
Inputs	Measured x_e, y_e, z_e, t, mass, diameter, air density, wind velocity direction cosines of sun spin profile. Estimated C_X, C_N, C_{Yp}, C_{lp}, C_m	Direction cosines t_1, t_2, t_3, Measured solar aspect angle, mass, moments of inertia, air density, wind velocity. Fitted p, q, V, V_w. Estimated C_N, C_{Yp}, C_m, C_{np}, C_{mq}
Outputs	Computed x_e, y_e, z_e, V, V_w, q, p Fitted C_X, C_{lp}	Computed σ vs t Fitted C_m, C_{Mq}, C_{np}

The swerve reduction also requires estimated values of axial force, normal force, Magnus force, spin profile, spin damping, and static moment coefficient. Such estimates can be obtained from the known aerodynamics of similar projectiles or from ballistic range measurements. The swerve reduction provides an output of axial force (related to drag) vs Mach number, the spin damping, and best fit computed values of position, velocity, dynamic pressure, and spin, all as functions of time.

The results of the swerve reduction are used as inputs to the angular reduction along with the solar aspect angle data from the yawsonde. The angular reduction also requires the physical properties of the projectile and the meteorological data as inputs. Initial estimated values of the static moment, Magnus moment, and damping moments must also be furnished. The angular reduction provides an output of best fitted values of solar aspect angle vs time plus the aerodynamic coefficients for static, Magnus and damping moments. Since the yawsonde data usually span extensive portions of the trajectory including many periods of precession and nutation, the data are fit sectionally. The results of sectional fitting are average values of the aerodynamic coefficients over the data intervals. The entire reduction procedure is summarized in Table 1.

Results and Discussion

The results of fitting for the translational and angular motion of a 155mm, M549 rocket assisted projectile are discussed below. This projectile was fired at Wallops Island Va., on March 11, 1971, at 2009 hr Universal Time from a twist tube. The muzzle velocity as determined by smear cameras was 321 m/sec. The gun elevation was 490 mils. The physical characteristics of the shell and the wind data are summarized in Table 2. An outline of the projectile is shown in Fig. 2.

Swerve Reduction

A swerve reduction was performed on the data obtained from an FPS-16 radar which tracked the projectile from 5.3 sec after launch to impact. In the swerve reduction the yaw

Table 2 Physical characteristics of shell and wind data

Weight	= 42.75 kg
Location of c.g.	= 32.21 cm from base
Axial moment of inertia	= 0.1477 kg-m^2
Transverse moment of inertia	= 1.8009 kg-m^2
Diameter	= 15.5 cm
Length	= 87.4 cm
Mean wind speed	= 12.8 m/sec
Mean wind direction	= 300° from true North

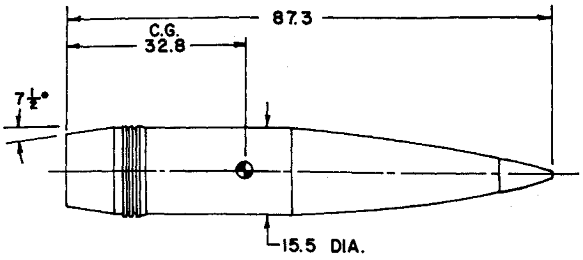

Fig. 2 Schematic of the 155mm, M549 rocket-assisted projectile.

of repose was estimated. Swerve reductions were performed both with and without wind effects. The results of the swerve reductions are given in Table 3. The probable error in each parameter is shown in parentheses below the parameter.

The muzzle velocity without wind effects agrees well with the smear camera value of muzzle velocity. The errors in the fit decrease when wind effects are taken into account. In order to accomplish the swerve reduction, values of normal force, Magnus force, and nonlinear axial force coefficients were estimated and held constant during the reduction.

The spin damping coefficient, C_{l_p}, was determined for the flight by matching a computed spin history to a flight spin history derived from the yawsonde data. In this manner, C_{l_p} was found to be $-0.025 \pm .001$.

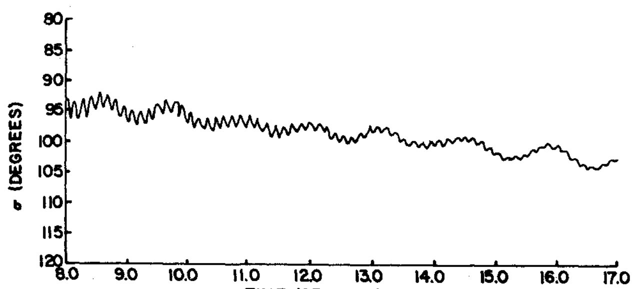

Fig. 3 Yawsonde flight data from an M549 projectile.

Table 3 Results of swerve reduction

	V_{Muzzle} m/s	V_{Impact} m/s	C_X	$C_{N\alpha}$	$C_{Y_{p\alpha}}$	$C_{X\alpha 2}$	Probable Error
Without wind	323 (0.12)	262 (0.12)	0.109 (0.0003)	1.6 [a]	−1.1 [a]	1.5 [a]	1.1 m
With wind	311 (0.09)	251 (0.09)	0.119 (0.0003)	1.6 [a]	−1.1 [a]	1.5 [a]	0.8 m

[a] Held constant.

Angular Reduction

A portion of the data from the yawsonde on board the projectile is shown in Fig. 3. The data are solar aspect angle vs time between 5.0 and 17.0 sec of the flight. The data clearly show a fast frequency and a slow frequency which can be identified as nutation and precession. The nutational amplitude damps out rather quickly while the precessional amplitude grows slowly. In order to obtain a reasonable value of damping moment at least several cycles of the precession must be included in the fit. The static moment will be the dominant factor.

A series of reductions were performed with variations in section length and data density. Data density refers to the number of data points per unit time included in the fit. Reductions were done with and without wind effects. Various combinations of linear and nonlinear coefficients were tried. The results of some reduction attempts are listed in Table 4. On some reductions a velocity dependent static moment coefficient, a_v, was sought. On other reductions, the ratio of moments of inertia was a parameter of the fit.

The coefficients were not substantially affected by changes in section length or data density as can be seen by comparing runs 2, 3, and 7. The maximum length used was 14.41 sec (run 7). For this length the probable error of fit was 0.31°. Runs 8–10 represent an attempt to extract nonlinear coefficients. A comparison between runs 1 and 2 shows the effects of wind. A change of 10% in static moment is observed and is considered significant. The changes in damping and Magnus moments are small and not significant.

The average total angle of attack for this flight was com-

Table 4 Aerodynamic coefficients derived from M549 yawsonde data.

Run No.	time interval sec	$C_{m\alpha}$ or a_0	C_{mq} or d_0	$C_{np\alpha}$ or b_0	a_3	d_2	b_3	a_v
1	5.0–13.93	4.233 (0.018)[a]	3.59 (0.88)	−0.796 (0.045)				0.0027
2[b]	5.0–13.93	4.642 (0.009)	2.80 (0.49)	−0.801 (0.030)				−0.0028
3	5.0–13.93	4.625 (0.015)	2.81 (0.79)	−0.803 (0.085)				−0.0025
4	5.0–15.29	4.679 (0.013)	1.65 (0.64)	−0.728 (0.035)				−0.0037
5	5.0–16.65	4.754 (0.0125)	1.81 (0.60)	−0.735 (0.029)				−0.0051
6	5.0–18.03	4.763 (0.0117)	3.65 (0.58)	−0.873 (0.025)				−0.0052
7	5.0–19.41	4.709 (0.012)	3.42 (0.60)	−0.869 (0.023)				−0.0043
8	5.0–19.41	4.56 (0.03)	3.78 (0.76)	−0.865 (0.024)	100.19 (22.9)			−0.0039
9	5.0–19.41	4.56 (.03)	3.78 (1.11)	−0.909 (.032)	113.65 (22.8)	1499.8 (1454.8)		−0.0039
10	5.0–19.41	4.68 (0.09)	.897 (1.6)	−0.49 (0.16)	8.23 (7.8)		−254.3 (231)	−0.0038

[a] Numbers in ( ) show probable error in parameters.
[b] The data density for run no. 2 was 80%. All other runs had only 20% data density.

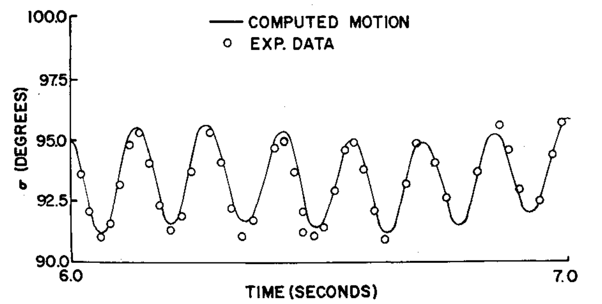

Fig. 4 A small data interval showing goodness of fit with only linear coefficients.

puted to be 1.72° which represents small amplitude angular motion. Runs 2–7 show the linear static moment coefficient varying from 4.64 to 4.76 at an average Mach number of 0.74 over this part of the flight. The fit for run 7 can be seen in Fig. 4 where the computed and measured solar aspect angle are plotted against time of flight. For the sake of short figures only a section of the fit interval in displayed.

In Fig. 5 the values of static moment coefficients from run 7 (linear) and run 10 (nonlinear) are compared to data from wind-tunnel tests performed on the M549 projectile. The total static moment at an average angle of attack of 1.72° is computed for an average Mach number of 0.74. Both the linear fit moment and the nonlinear fit moment agree well with

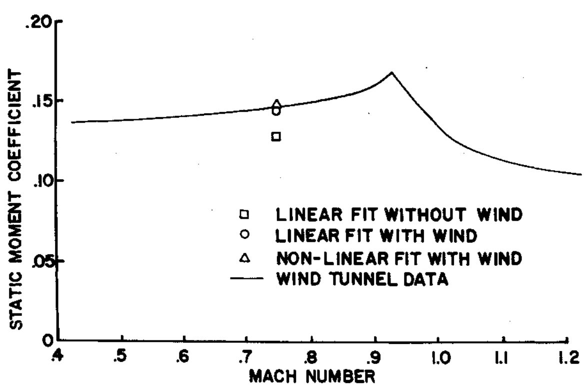

Fig. 5 A comparison between C_m obtained from yawsonde and wind-tunnel tests.

wind-tunnel results. To show the effect of not including wind, the static moment result of run 1 is also shown. The point is noticeably away from the wind-tunnel data.

Wind-tunnel Magnus moment measurements at the above values of Mach number and angle of attack give a $C_{n_p} = 0.0125$ while the Magnus moment computed from run 7 is -0.0124. The damping moment coefficient from wind-tunnel tests is -5.1 while run 7 shows $+3.4$. In light of the yawsonde data shown in Fig. 3, it would seem that Magnus and damping moments cannot be readily extracted from small amplitude motion where the nutational mode dominates the data.

Conclusions

A technique of numerical integration has been developed and applied to experimental radar and yawsonde data. The effects of wind and velocity dependent coefficients can readily be included in the method. Position-time radar data are readily reduced to aerodynamic coefficients with satisfactory results. Yawsonde data from a flight test of the M549 projectile easily yielded the static moment coefficient. Magnus and damping moment coefficients are difficult to extract from small amplitude, nutation-dominated motion. The capability of extracting three-dimensional angular motion from two-dimensional yawsonde data has been demonstrated.

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